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Engineering Notes

Micro Air Vehicle Trajectory Planning in Winds

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I. Introduction

MICRO aerial vehicles (MAVs) are small aerial vehicles, generally with dimensions on the order of 1 ft (or 0.3 m). Their small sizes often imply that their flight speeds are also small, roughly on the order of 20–60 km/h. MAVs come in a wide variety of forms, such as fixed wing, flapping wing, and rotary wing [1]. They are finding increasingly more applications in such areas as military missions, reconnaissance of hazardous or remote areas, and monitoring of indoor areas.

Because of their small sizes and light weights, MAV flight trajectories are significantly susceptible to winds in general. For example, it is possible for the wind to be so strong that the MAV is unable to advance in certain directions. (This problem can be formulated in terms of the controllability of a point and its associated reachable set, see [2].) It is therefore necessary to develop a systematic approach to MAV trajectory generation that addresses the characteristic issues of MAV flights in winds. Specifically, the following three points must be considered:

1) Target points may not always be feasible due to winds.
2) Wind profiles may have significant effects on fuel costs (both desirable and undesirable).
3) Trajectory generation must always yield a feasible solution due to the absence of human operators.

The main goal of this note is to study meaningful trajectory generation problem formulations for MAV trajectories in winds, to address all three of the issues discussed above that characterize MAV flights. Specifically, MAV flights in winds are formulated as nonlinear optimal control problems, with proper constraints on states and controls. In particular, the reaching of a target point is enforced via a penalty term in the performance index. Thus, while a target point is not necessarily reached, a flyable trajectory that is optimal under prevailing conditions is obtained. Both constant and position dependent wind is considered.

II. Equations of Motion and MAV Models

For the purposes of trajectory optimization, MAV motions can be adequately represented by a dynamic point-mass model. This Note is concerned with travel trajectories that may be assumed to be in horizontal planes for most practical purposes. The corresponding equations of motion for MAV flights in the horizontal plane are summarized below for convenience [3], where zero vertical wind is assumed:

$$\dot{V} = (T - D)/m - \dot{W}_x \sin\Psi - \dot{W}_y \cos\Psi \tag{1}$$

$$V\dot{\Psi} = L \sin\mu/m + \dot{W}_x \cos\Psi - \dot{W}_y \sin\Psi \tag{2}$$

$$\dot{x} = V\cos\Psi + W_x = V_I \sin\Psi_I \tag{3}$$

$$\dot{y} = V\sin\Psi + W_y = V_I \cos\Psi_I \tag{4}$$

where V and V_I are the airspeed and ground speed, respectively; T is thrust; L and D denote lift and drag, respectively; W_x and W_y are the wind speeds in the x and y directions; Ψ is the heading angle; and μ is the bank angle. For level flights with zero vertical wind,

$$L\cos\mu = mg \tag{5}$$

In these equations, the UAV mass m is assumed constant. The lift and drag forces are

$$L = \tfrac{1}{2}\rho V^2 S C_L, \qquad D = \tfrac{1}{2}\rho V^2 S C_D \tag{6}$$

where C_D and C_L, are the drag and lift coefficients, respectively, S is the wing area, and ρ is the density of the air. The drag coefficient is modeled by the parabolic drag polar [4]:

$$C_D = C_{D_0} + KC_L^2 \tag{7}$$

where the induced drag factor K can be determined from the aerodynamic efficiency $E_{\max}$ and the zero-lift drag coefficient C_{D_0} as

$$K = \frac{1}{4E_{\max}^2 C_{D_0}} \tag{8}$$

For good numerical efficiency in the optimization process, speed variables are normalized by a characteristic speed V_c, distances by V_c^2/g, time by V_c/g, and thrust by the weight mg (see [3]). A normalized variable is denoted by an overbar (e.g., $\bar{W}_x = W_x/V_c$).

III. Classification of Air Vehicle Flights in Winds

In general, the effect of winds on aircraft flights is highly dependent on the overall speed of the aircraft. To facilitate the studies of optimal air vehicle trajectory planning in winds, it is useful to systematically characterize and classify the significance of wind based on typical airspeeds of aircraft. Specifically W_x and W_y in Eqs. (1–4) may be expressed in terms of their magnitude and W_m and Ψ_W, respectively:

$$W_x = W_m \sin\Psi_W, \qquad W_y = W_m \cos\Psi_W \tag{9}$$

and the ratio of the wind-speed magnitude to the airspeed of the aircraft is defined as

$$\sigma = W_m/V \tag{10}$$

In what follows, the ratio σ is used to characterize the importance of wind in trajectory generations for various types of flight.

A. Supersonic Flight

When airspeeds are larger than wind speeds by several orders of magnitude, it may be expected that the effect of wind is generally

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very small. In particular, in the case of supersonic flights in typical winds, the ratio σ is on the order of 0.01–0.05. Given the fact that knowledge of the wind itself will typically contain larger uncertainties, the effect of wind can generally be neglected.

B. High-Speed Subsonic Flight:

This type of flight may be characteristic of commercial transport aircraft, where typical values of σ will be in the range of 0.05–.1. In this case, winds can significantly affect flight times between two points and the corresponding fuel requirements. Nonetheless, wind effects are generally not so large as to affect the reachability of any target point.

C. Medium-Speed Subsonic Flight

Light single-engine aircraft and large UAVs typically fly at speeds that correspond to values of σ in the range of 0.5–1. In these cases, winds become critically important in shaping flight trajectories. While it is still generally possible for such aircraft to arrive at desired destinations in any given direction, there will occasionally be situations of strong wind (for example, due to wind gusts), where $\sigma > 1$ for short periods of time, when the aircraft cannot advance against the wind direction in the inertial frame.

D. Low-Speed Flight

MAVs typically fly at speeds close to or even smaller than typical wind speeds. The corresponding values of σ may be in the range of 1–10, with typical Reynolds numbers that are less than 2×10^5. Their flight trajectories are significantly affected by winds. The wind terms in Eqs. (1–4) are therefore of equal importance to the airspeed V. In these cases, winds directly affect the feasibility of a flight trajectory to a specified target point and it can no longer be assumed that the aircraft can fly in any given direction in the inertial frame, due to prevailing winds.

E. Wandering Flights

Average speeds of very small or tiny air vehicles are in general much smaller than prevailing wind speeds, and typical values of σ may be greater than 10. Reynolds numbers in this case may be on the order of 10,000 (similar to insect flight). In Eqs. (3) and (4), the wind terms W_x and W_y can dominate the solutions. Therefore, any significant large scale motions of these vehicles with respect to the ground are a result of winds.

IV. Mathematical Formulations of Travel Trajectory Planning

Travel trajectory planning for a MAV is concerned with flights between two specified points in space. This is in contrast to loitering flights, for example, in which a MAV is simply required to stay within a specified area.

The performance parameters of interest in travel trajectory planning include the time of flight, the closeness to the target point, and the amount of fuel consumption. Because of the possibility that a target point may not be reachable at a given time, it is not feasible to include the target-point satisfaction as a hard constraint. Instead, it may be included as a soft constraint through a penalty term in the performance index. In addition, it is difficult to assume a fixed flight time, because of the strong effects of the winds. Therefore, an optimal trajectory planning problem for low-speed flights such as those of MAVs may be formulated as follows,

$$\min_{T,\mu;t_f} I = \| \mathbf{x}(t_f) - \mathbf{x}_g \| + K_t t_f + K_f \int_0^{t_f} TV \, \mathrm{d}t \tag{11}$$

subject to Eqs. (1–4). In this problem, where t_f is the final time, K_f and K_t are constants, $\mathbf{x}(t_f)$ is a vector of state variables at that final time, and $\mathbf{x}_g$ is the desired final value of $\mathbf{x}$. This approach gives the flexibility of choosing the relative importance of fuel costs and time of flight to match the requirements of a given specific mission.

In addition, the MAV needs to satisfy general path constraints arising from operational and performance limitations. Constraints on trajectory state and control variables include

$$\bar{V}_{\min} \le \bar{V} \le \bar{V}_{\max}, \qquad |\mu| \le \mu_{\max}, \qquad 0 \le \bar{P} \le \bar{P}_{\max} \tag{12}$$

where P is the nondimensional power generated, and where constraints on the lift coefficient, $C_{L_{\min}} \le C_L \le C_{L_{\max}}$, become

$$\frac{1}{\bar{\rho} C_{L_{\max}}} \le \bar{V}^2 \cos\mu \le \infty \tag{13}$$

There are also bounds on the time rates of trajectory control variables:

$$|\dot{\mu}| \le \dot{\mu}_{\max}, \qquad |\bar{T}'| \le \bar{T}'_{\max} \tag{14}$$

In level flights, a load factor limit can be enforced through the bound on the bank angle μ. Because

$$n = \frac{L}{W} = \bar{\rho}\bar{V}^2 C_L = \frac{1}{\cos\mu} \le n_{\max} \tag{15}$$

it follows that

$$|\mu| \le \mu_{\max} = \cos^{-1}\left(\frac{1}{n_{\max}}\right) \tag{16}$$

V. Numerical Studies

In this section, optimal trajectories are presented that are obtained numerically for a MAV using the problem formulation given in Eq. (11). The numerical solutions are based on a collocation approach [5–7], in which both state and control variables are parameterized. The time interval of optimal trajectory solutions is divided into N equally spaced subintervals. In the results presented below, $N = 31$ is used. It is further assumed that the normalizing velocity is $V_c = 108$ km/h (100 ft/s). The zero-lift drag coefficient is assumed to be $C_{D0} = 0.025$, and maximum lift-to-drag ratio is taken to be $(L/D)_{\max} = 8.0$, respectively. Also, recalling that an overbar denotes a normalized quantity, $\bar{V}_{\min} = 0.25$, $\bar{V}_{\max} = 2.5$, $\bar{T}_{\max} = 0.25$, and $\mu_{\max} = 0.25$.

The first case considered is wind that is constant in both time and position. Figure 1 illustrates the effects of the magnitude of the wind, as well as the initial position of the MAV, on the possibility for trajectories to reach the target. The coefficients in the cost function are here taken to be $K_f = 0.1$ and $K_t = 0.1$. The dashed lines in this figure correspond to $\bar{W} = 0.5$ (which is about one-fifth of the maximum velocity), while the solid lines correspond to $\bar{W} = 4.0$, which is about 50% larger than the maximum velocity. As would be expected, when the wind is weak it is possible to reach the target from

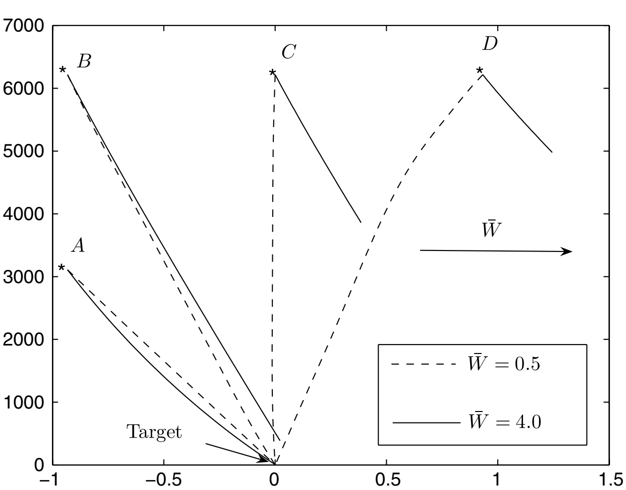

Fig. 1 Effect of wind magnitude on optimal MAV trajectories in constant wind.

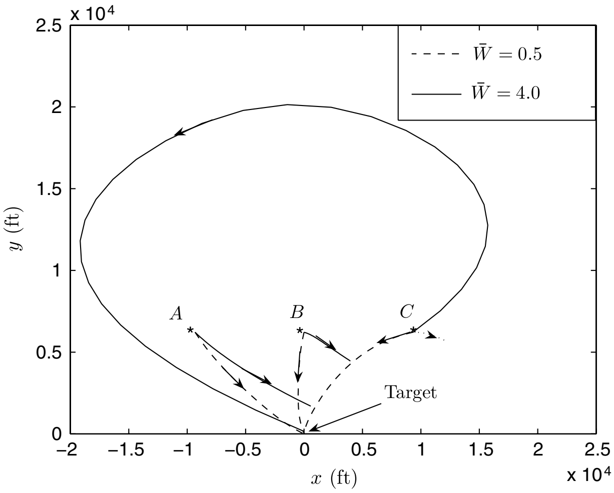

Fig. 2 Exploitation of global wind pattern for optimal MAV trajectory.

all the starting points. In the case of $\bar{W} = 4.0$, only the trajectory starting from point A reaches the target. It is easy to verify that all the other trajectories lie on the lower boundaries of the respective starting point's feasibility region at slopes of roughly -0.6.

As an example of position-varying wind, the following wind profile is considered:

$$\bar{W}_x = \bar{W}_m(40 - \bar{y})/20, \qquad \bar{W}_y = 0 \tag{17}$$

The coefficients in the objective function are taken to be $K_f = 0.1$ and $K_t = 0.1$. Figure 2 shows a set of trajectories from points A, B, and C, respectively, at $(\bar{x}, \bar{y}) = (-30, 20)$, $(0, 20)$, and $(30, 20)$. In the figure the dashed trajectories correspond to $\bar{W} = 0.5$, i.e., less than the maximum airspeed of the MAV. It is seen that these trajectories all reach the target, regardless of initial point. On the other hand, when $\bar{W} = 4.0$, which is greater than the maximum airspeed, the optimal trajectories starting at points A and B do not reach the target. However, the trajectory starting at point C makes a detour around the strong wind region through areas of weaker wind (where it flies against the wind) to finally reach the target from the upwind direction, as required. Note that the trajectories obtained depend highly on the chosen values of K_f and K_t. For example, for $K_f = 0.5$ and $K_t = 0.5$, the trajectory that starts at point C does not reach the target (dotted curve).

VI. Conclusions

The continued technological advancement of MAVs presents a potentially versatile tool for wide-ranging applications in environmental health, law enforcement, border security, etc. A major challenge for the successful use of MAVs due to their slow-flying nature, is caused by the presence of generally unpredictable winds. In this Note, a formulation of optimal trajectory planning problem that always results in a flyable solution for the MAV is presented. In this formulation, one primarily seeks to minimize the difference between the final state of the MAV (that may include position and velocity) to the target state, as opposed to specifying the target state as the final state. Thus, a solution that reaches the target may or may not be obtained.

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